

Control Lec II

observer design

* The observer is used to estimate the unmeasurable states.

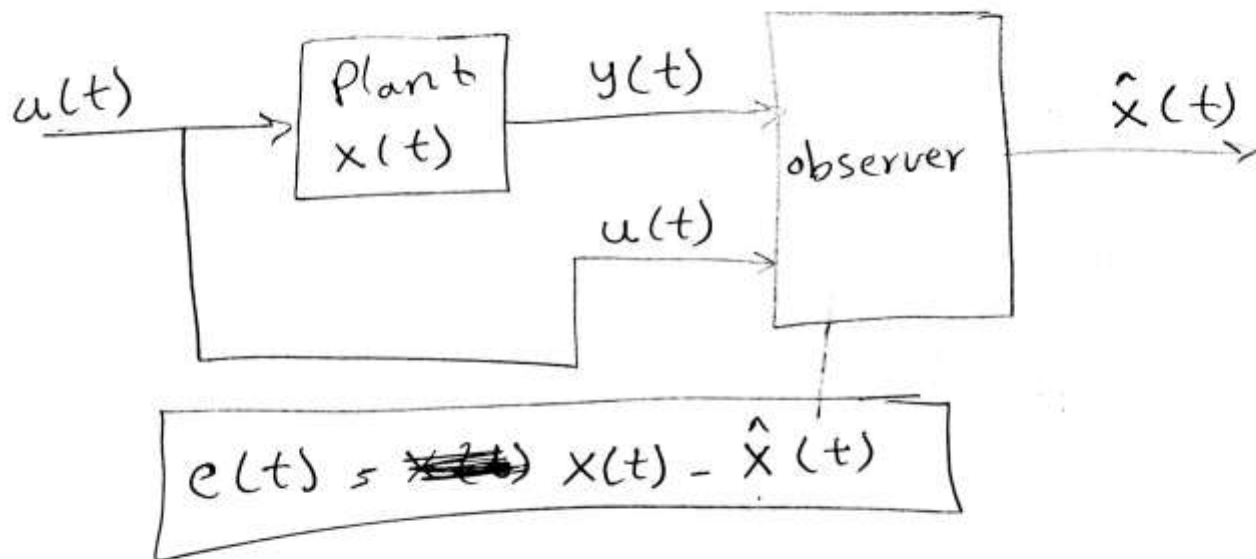
observer

Full order

(estimation is done for all states)

Reduced order

(estimation is done for some states not for all states)



* The observer eqn

$$\dot{\hat{x}}(t) = (A - Gc) \hat{x}(t) + Bu(t) + G y(t)$$

* The design is to find gain matrix "G"

المهمة (design) هي إيجاد specs \rightarrow G

* To obtain the gain G , the design specs. will be contained the required transient response for the observer (given desired poles)

[1] desired ch. equation $\alpha_o(s)$

$$\alpha_o(s) = (s - p_1)(s - p_2) \dots \quad (1)$$

$p_1, p_2 \rightarrow$ desired poles (given)

desired ch. equation in terms of G

$$|sI - (A - Gc)| = 0 \quad (2)$$

$$[2] \quad G = \alpha_o(A) M_o^{-1} \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix}$$

$M_o \Rightarrow$ observability Matrix

$$\alpha_o(A) = \alpha_o(s) \Big|_{s \rightarrow A} \quad [2]$$

Ex Design an observer for the following system:-

$$\dot{x} = \begin{pmatrix} 2 & 0 \\ 1 & 0 \end{pmatrix} x(t) + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u(t)$$

$$y(t) = (1 \quad -1) x(t)$$

The observer poles locate at $-1, -2$

Sol

1) desired ch-equation for the observer:-

$$\alpha_o(s) = (s+1)(s+2) = 0$$

$$\boxed{\alpha_o(s) = s^2 + 3s + 2}$$

$$\alpha_o(A) = A^2 + 3A + 2I$$

$$\alpha_o(A) = \begin{pmatrix} 2 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 1 & 0 \end{pmatrix} + 3 \begin{pmatrix} 2 & 0 \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 0 \\ 2 & 2 \end{pmatrix} + \begin{pmatrix} 6 & 0 \\ 3 & 0 \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \quad \begin{pmatrix} +2 \\ +2 \end{pmatrix}$$

$$\alpha_0(A) = \begin{pmatrix} 12 & 0 \\ 5 & 2 \end{pmatrix}$$

$$2) M_0 = \begin{pmatrix} C \\ CA \end{pmatrix}$$

$$CA = (1 \quad -1) \begin{pmatrix} 2 & 0 \\ 1 & 0 \end{pmatrix} = (1 \quad 0)$$

$$M_0 = \begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}$$

$$M_0^{-1} = \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix}$$

$$G = \alpha_0(A) M_0^{-1} \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 12 & 0 \\ 5 & 2 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$G = \begin{pmatrix} 12 \\ 7 \end{pmatrix}$$